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Escola de Pós-Graduação  
em Economia

## Ensaaios Econômicos

Escola de

Pós Graduação

em Economia

da Fundação

Getulio Vargas

Nº 593

ISSN 0104-8910

***The Asymmetric Behavior of the U.S. Public  
Debt.***

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**Julho de 2005**

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# The Asymmetric Behavior of the U.S. Public Debt.

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June 29, 2005

## Abstract

In this paper we re-analyze the question of the U.S. public debt sustainability by using a quantile autoregression model. This modeling allows for testing whether the behavior of U.S. public debt is asymmetric or not. Our results provide evidence of a *band of sustainability*. Outside this band, the U.S. public debt is unsustainable. We also find fiscal policy to be adequate in the sense that occasional episodes in which the public debt moves out of the band do not pose a threat to long run sustainability.

- JEL Classification: C22, E60, H60;
- Keywords: Fiscal Policy, Long-Run Sustainability, Quantile autoregression.

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# 1 Introduction

For decades, a lot of effort has been devoted to investigate whether long-lasting budget deficits represent a threat to public debt sustainability. Hamilton and Flavin (1986) was one of the first studies to address this question testing for the non-existence of ponzi scheme in public debt. They conducted a battery of tests using data from the period 1962-84 and assuming a fixed interest rate. Their results indicate that the government intertemporal budget constraint holds. In a posterior work, Wilcox (1989) extends Hamilton and Flavin's work by allowing for stochastic variation in the real interest rate. His focus was on testing for the validity of the present-value borrowing constraint, which means that the public debt will be sustainable in a dynamically efficient economy<sup>1</sup> if the discounted public debt is stationary with unconditional mean equal to zero.

An important and common feature in the aforementioned studies is the underlying assumption that economic time series possess symmetric dynamics. In recent years, considerable research effort has been devoted to study the effect of different fiscal regimes on long-run sustainability of the public debt. For instance, Davig (2004) uses a Markov-switching time series model to analyze the behavior of the discounted U.S. Federal debt. The author uses an extended version of Hamilton and Flavin (1986) and Wilcox (1989) data and identifies two fiscal regimes: in the first one the discounted Federal debt is expanding whereas it is collapsing in the second one. He concludes that although the expanding regime is not sustainable, it does not pose a threat to the long run sustainability of the discounted U.S. federal debt. Arestis et al. (2004) model the U. S. government deficit per capita using a threshold autoregressive model. Two fiscal policy regimes are identified by the extent of the semi-annual change in the deficit. Using quarterly deficit data from the period 1947:2 to 2002:1, they find evidence that the U.S. budget deficit is sustainable in the long run and that economic policy makers only intervene to reduce budget deficit when it reaches a certain threshold, deemed to be unsustainable.

In this paper we reanalyze the question of public debt sustainability using the quantile autoregression model recently developed by Koenker and Xiao (2004a). In this new model, the autoregressive coefficient may take different values (possibly unity) over different quantiles of the innovation process. Some forms of the quantile autoregressive (QAR) model can exhibit unit-root like behavior, but with occasional episodes of mean reversion sufficient to insure stationarity. We use an extension of Hamilton and Flavin (1986) and Wilcox (1989) data, provided by Davig (2004). As in Wilcox (1989), our data set allows for stochastic interest rate as we deal with the discounted public debt series. We believe that this modelling is ideal for the following reasons:

- it allows us to test the hypothesis that discounted U.S. Federal debt exhibits asymmetric dynamic;
- it is possible to test for both global and local sustainability. The latter allows us to identify fiscal policies (trajectories of public debt) that are not

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<sup>1</sup> Abel et al. (1996) provides evidence that the U.S. economy is dynamically efficient.

consistent with public debt sustainability in the sense that if they were allowed to persist indefinitely, would eventually violate intertemporal restrictions;

- in contrast to the threshold autoregressive model used by Arestis et al. (2004), we do not have to consider only two regimes and split the sample points into them. This avoids identification problems in short samples;

- unlike Davig (2004) that assumes an autoregressive process with just one lag and a deterministic first-order autoregressive coefficient, we consider the possibility that the discounted U.S. Federal debt series can be represented by a autoregressive process with  $p$  lags,  $p \geq 1$ , in which all autoregressive coefficients are random. In the Davig (2004) approach, only the intercept is allowed to change across regimes and the process is assumed to be local as well as globally stationary. Consequently, the author does not test for local and global stationarity. Thus, the tests for local and global public debt sustainability in Davig (2004) consist in verifying if the unconditional mean of the debt process is not significantly different from zero. In this paper, we will investigate if both conditions (stationarity and zero unconditional mean) holds locally as well as globally;

- it permits to identify the dynamic of the discounted U.S. Federal debt at different quantiles. Hence, we can easily forecast debt values that should be avoided by policy makers if they were interested in keeping public debt sustainable. We believe that policy makers might be interested in using such forecasts as guidance instruments for monetary and fiscal policies;

- in contrast to both Davig (2004) and Arestis et al. (2004), our estimation procedure does not require a distribution specification for the innovation process, which makes our approach robust against distribution misspecification. Indeed, estimation in Davig (2004) and Arestis et al. (2004) are based on maximum likelihood<sup>2</sup>, which requires distribution specification. Quantile regression method is robust to distributional assumptions, a property that is inherited from the robustness of the ordinary sample quantiles. Moreover, in quantile regression, it is not the magnitude of the dependent variable that matters but its position relative to the estimated hyperplane. As a result, the estimated coefficients are not sensitive to outlier observations. Such property is especially attractive in our application since public debt may be contaminated by outlier observations coming from periods of war, oil shocks and political turmoil.

Our results expand previous findings on public debt sustainability reported in the literature. Unlike Davig (2004), the quantile estimates provide evidence of what we have called *band of sustainability*: the discounted U.S. federal debt is sustainable for values inside the band, but becomes unsustainable when it jumps out of the band. In other words, the fiscal policy follows three different dynamics according to the level of the discounted U.S. Federal debt: (1) for values around (and inclusive) the median, fiscal policy is sustainable, ruling out the possibility of permanent budget deficits; (2) when public debt attains high values (high quantiles), fiscal policy turns out to be unsustainable, with the

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<sup>2</sup>Estimation in Arestis et al (2004) is carried out using ordinary least squares which is numerically equivalent to the maximum likelihood estimator under Gaussian assumptions.

discounted debt not converging to zero, even though it follows a stationary path; (3) when the debt is low (low quantiles), fiscal policy is unsustainable, presenting both nonstationarity and non-zero (and negative) unconditional mean. Despite the occasional episodes in which the discounted U.S. federal debt moves out of the band of sustainability, our results indicate that the U.S. debt is globally sustainable, i.e., the periods in which public debt is too high or too low do not seem to persist forever. This suggests that the U.S. government has been committed with long-run sustainability of the public debt, bringing it back to its sustainability band whenever it moves out.

This study is organized as follows. Next section states formally the questions addressed in this paper. Sections 3 and 4 describe the econometric model and inference methods used in our analysis. Section 5 presents the results, and Section 6 summarizes the main conclusions.

## 2 Theory: the present-value borrowing constraint

The accumulation of government debt follows the equation:

$$b_t = (1 + r_{t-1})b_{t-1} - s_t \quad (1)$$

where:

$b_t$  is the market value of the government debt in constant dollars;

$r_{t-1}$  is the ex-post real interest rate

$s_t$  is the noninterest surplus in constant dollars.

Let  $q_t$  be the real discount factor from period  $t$  back to period zero, defined by:

$$q_t = \prod_{j=0}^{t-1} \frac{1}{(1 + r_j)}, \quad t = 1, 2, \dots$$

and  $q_0 = 1$

thus,  $q_t$  is a function of  $(r_0, r_1, \dots, r_{t-1})$  and it is known at period  $t$ .

Using  $q_t$  it is possible to rewrite equation (1) with each variable discounted to period zero:

$$\begin{aligned} q_t b_t &= q_t (1 + r_{t-1}) b_{t-1} - q_t s_t \\ q_t b_t &= q_{t-1} b_{t-1} - q_t s_t \end{aligned}$$

Let  $B_t := q_t b_t$  be the discounted value of debt in period  $t$  back to period zero, and  $S_t := q_t s_t$  be the discounted value of the surplus in period  $t$  back to period zero. Then:

$$B_t = B_{t-1} - S_t.$$

Solving equation by recursive forward substitution leads to:

$$B_t = B_{t+N} + \sum_{j=1}^N S_{t+j}. \quad (2)$$

As observed by Hamilton and Flavin, this equation are simple manipulations of an accounting identity and thus is not a point of serious controversial. "What is of economic interest (and subject in principle to empirical refutation) is what creditors expect to happen to the first term in (2) as  $N$  gets large." If that term is expected to go to zero as  $N$  goes to infinity, then the discounted value of the debt is expected to be equal to the discounted value of the sum of all future noninterest surpluses:

$$\begin{aligned} B_t &= \lim_{N \rightarrow \infty} E_t B_{t+N} + \sum_{j=1}^{\infty} E_t S_{t+j} \\ \lim_{N \rightarrow \infty} E_t B_{t+N} &= 0 \Rightarrow \\ \Rightarrow B_t &= \sum_{j=1}^{\infty} E_t S_{t+j} \end{aligned}$$

where the last equality is called the present-value borrowing constraint (PVBC). An obvious implication of PVBC is that governments cannot run deficits forever, but the meaning of this constraint is more subtle and weaker in terms of fiscal policy restriction: even though governments can run deficits in some periods; these deficits must be compensated with sufficiently large surpluses in the future.

Wilcox (1989) suggests a natural concept of sustainability: a sustainable fiscal policy is one that would be expected to generate a sequence of deficits and surpluses such that the present-value borrowing constraint would hold.

But, given a time series of discounted public debt, how can we distinguish between a sustainable fiscal policy from an unsustainable one? Obviously, the answer passes through the forecast trajectory for  $B_{t+N}$ . For the PVBC to hold, the forecast trajectory must converge to zero. Convergence is related to stationarity of the series, but even if the series is stationary, it can still converge to a constant number different from zero. Therefore, in order to guarantee sustainability it is also necessary that the unconditional mean of the process equals zero.

The technical issues about testing for public debt sustainability are covered in Sections 3 and 4, where we introduce the econometric model and discuss inference methods.

### 3 The Econometric Model

We investigate the presence of asymmetric dynamic in the discounted U.S. federal debt using the quantile autoregression model introduced by Koenker

and Xiao (2002, 2004a, 2004b). This model is a random coefficient time series model whose autoregressive coefficients parameters are functionally dependent and may vary over the quantiles  $\tau \in (0, 1)$ . Therefore, it sheds light on the asymmetric behavior of the U.S. public debt and provides means to test the null hypothesis of symmetric dynamic in such series.

We are also able to test for both global and local sustainability, with global sustainability referring to a set of quantiles and local sustainability analyzing the behavior of U.S. public debt at a fixed quantile. The latter allows us to identify fiscal policies (trajectories of the public debt) that are not consistent with public debt sustainability in the sense that if they were allowed to persist indefinitely, they would eventually violate intertemporal restrictions.

### 3.1 The Quantile Autoregression Model

Let  $\{U_t\}$  be a sequence of iid standard uniform random variables, and consider the  $p$ th order autoregressive process,

$$y_t = \theta_0(U_t) + \theta_1(U_t)y_{t-1} + \dots + \theta_p(U_t)y_{t-p} \quad (3)$$

where  $\theta_j$ 's are unknown functions  $[0, 1] \rightarrow \mathbb{R}$  that we will want to estimate. We will refer to this model as the  $QAR(p)$  model.<sup>3</sup>

The  $QAR(p)$  model (3) can be reformulated in a more conventional random coefficient notation as,

$$y_t = \mu_0 + \beta_{1,t}y_{t-1} + \dots + \beta_{p,t}y_{t-p} + u_t \quad (4)$$

where

$$\begin{aligned} \mu_0 &= E\theta_0(U_t) \\ u_t &= \theta_0(U_t) - \mu_0 \\ \beta_{j,t} &= \theta_j(U_t), \quad j = 1, \dots, p \end{aligned}$$

Thus,  $\{u_t\}$  is an iid sequence of random variables with distribution  $F(\cdot) = \theta_0^{-1}(\cdot + \mu_0)$ , and the  $\beta_{j,t}$  coefficients are functions of this  $u_t$  innovation random variable.

As seen in Section 2, our sustainability concept involves an analysis of the unconditional mean of  $y_t$ . Koenker and Xiao (2004b) give an analytical representation of the unconditional mean of  $y_t$ . In other words, they show that if the time series  $y_t$ , given by (4), is covariance stationary and satisfies a central limit theorem, then

$$\frac{1}{\sqrt{n}} \sum_{t=1}^n (y_t - \mu_y) \Rightarrow N(0, \omega_y^2),$$

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<sup>3</sup>More on regularity conditions underlying model (3) are found in Koenker and Xiao (2004a)



where

$$\begin{aligned}\mu_y &= \frac{\mu_0}{1 - \sum_{j=1}^p \beta_j} \\ \omega_y^2 &= \lim \frac{1}{n} E \left[ \sum_{t=1}^n (y_t - \mu_y) \right]^2 \\ \beta_j &= E(\beta_{j,t}), \quad j = 1, \dots, p.\end{aligned}$$

Therefore, the unconditional mean of  $y_t$ ,  $\mu_y$ , will equal to zero when  $E\theta_0(U_t) = \mu_0 = 0$ .

$QAR(p)$  models can play a useful role in expanding the territory between classical stationary linear time series and their unit root alternatives. To illustrate this in the  $QAR(1)$  case, consider the following example

$$y_t = \beta_{1,t} y_{t-1} + u_t$$

with  $\beta_{1,t} = \theta_1(U_t) = \min\{\gamma_0 + \gamma_1 U_t, 1\} \leq 1$  where  $\gamma_0 \in (0, 1)$  and  $\gamma_1 > 0$ . In this model, if  $U_t > \frac{(1-\gamma_0)}{\gamma_1}$  then the model generates  $y_t$  according to the unit root model, but for smaller realizations of  $U_t$  we have mean reversion tendency. Thus, the model exhibits a form of asymmetric persistence in the sense that sequences of strongly positive innovations tend to reinforce its unit root like behavior, while occasional negative realizations induce mean reversion and thus undermine the persistency of the process. Therefore, it is possible to have locally nonstationary time series being globally stationary.

An alternative form of the model (4) widely used in economic applications is the ADF(augmented Dickey-Fuller) representation :

$$y_t = \mu_0 + \alpha_1 y_{t-1} + \sum_{j=1}^{p-1} \alpha_{j+1} \Delta y_{t-j} + u_t \quad (5)$$

where, corresponding to (3),

$$\begin{aligned}\alpha_{1,t} &= \sum_{s=1}^p \beta_{s,t} \text{ and} \\ \alpha_{j+1,t} &= - \sum_{s=j+1}^p \beta_{s,t}, \quad j = 2, \dots, p.\end{aligned}$$

In this model, the autoregressive coefficient  $\alpha_{1,t}$  plays an important role in measuring persistency in economic and financial time series. Under regularity conditions, if  $\alpha_{1,t} = 1$ ,  $y_t$  contains a unit root and is persistent; and if  $|\alpha_{1,t}| < 1$ ,  $y_t$  is stationary.

Notice that Equations (3), (4) and (5) are equivalent representations of our econometric model. Each representation is convenient for the inference analysis conducted in next section.

### 3.2 Estimation

Provided that the right hand side of (3) is monotone increasing in  $U_t$ , it follows that the  $\tau$ th conditional quantile function of  $y_t$  can be written as,

$$Q_{y_t}(\tau \mid y_{t-1}, \dots, y_{t-p}) = \theta_0(\tau) + \theta_1(\tau) y_{t-1} + \dots + \theta_p(\tau) y_{t-p} \quad (6)$$

or somewhat more compactly as,

$$Q_{y_t}(\tau \mid y_{t-1}, \dots, y_{t-p}) = x_t^T \theta(\tau),$$

where  $x_t = (1, y_{t-1}, \dots, y_{t-p})^T$ . The transition from (3) to (6) is an immediate consequence of the fact that for any monotone increasing function  $g$  and a standard uniform random variable,  $U$ , we have:

$$Q_{g(U)}(\tau) = g(Q_U(\tau)) = g(\tau),$$

where  $Q_U(\tau) = \tau$  is the quantile function of  $U_t$ .

Analogous to quantile estimation, quantile autoregression estimation involves the solution to the problem:

$$\min_{\{\theta \in \mathbb{R}^{p+1}\}} \sum_{t=1}^n \rho_\tau(y_t - x_t^T \theta) \quad (7)$$

where  $\rho_\tau$  is defined as in Koenker and Basset(1978):

$$\rho_\tau(u) = \begin{cases} \tau u, & u \geq 0 \\ (\tau - 1)u, & u < 0 \end{cases}.$$

The quantile regression method is robust to distributional assumptions, a property that is inherited from the robustness of the ordinary sample quantiles. Moreover, in quantile regression, it is not the magnitude of the dependent variable that matters but its position relative to the estimated hyperplane. As a result, the estimated coefficients are not sensitive to outlier observations.

### 3.3 Hypothesis Testing

#### 3.3.1 Autoregressive Order Choice

Equation (4) gives our  $p$ th order quantile autoregression model. We now discuss how to choose the optimal lag length  $p$ . We follow Koenker and Machado (1999) in testing for the null hypothesis of exclusion for the  $p$ th control variable.

$$H_0 : \theta_p(\tau) = 0, \text{ for all } \tau \in T \quad (8)$$

for some index set  $T \subset (0, 1)$ .

Let  $\hat{\theta}(\tau)$  denote the minimizer of

$$\hat{V}(\tau) = \min_{\{\theta \in \mathbb{R}^{p+1}\}} \sum \rho_\tau(y_t - x_t^T \theta)$$

where  $x_t^T = (1, y_{t-1}, y_{t-2}, \dots, y_{t-p})^T$  and  $\tilde{\theta}(\tau)$  denotes the minimizer for the corresponding constrained problem without the  $p$ th autoregressive variable, with

$$\tilde{V}(\tau) = \min_{\{\theta \in \mathbb{R}^p\}} \sum \rho_\tau(y_t - x_{1t}^T \theta)$$

where  $x_{1t}^T = (1, y_{t-1}, y_{t-2}, \dots, y_{t-(p-1)})^T$ . Thus,  $\hat{\theta}(\tau)$  and  $\tilde{\theta}(\tau)$  denote the unrestricted and restricted quantile regression estimates.

Machado e Koenker(1999) states that we can the null hypothesis (8) using a related version of the Likelihood process for a quantile regression with respect to *several* quantiles. Suppose that the  $\{u_t\}$  are iid but drawn from some distribution, say,  $F$ , and satisfying some regularity conditions. The LR statistics at a fixed quantile is derived as follows:

$$L_n(\tau) = \frac{2 \left( \tilde{V}(\tau) - \hat{V}(\tau) \right)}{\tau (1 - \tau) s(\tau)} \quad (9)$$

where  $s(\tau)$  is the sparsity function

$$s(\tau) = \frac{1}{f(F^{-1}(\tau))}.$$

The sparsity function, also termed the quantile-density function, plays role of a nuisance parameter, whose estimation we discuss in the Appendix.

We want to carry out a joint test about the significance of the  $p$ th autoregressive coefficient with respect to a set of quantiles  $T$  (not only at fixed quantile). Koenker and Machado (1999) suggests using the Kolmogorov-Smirnov type statistics for the joint test:

$$\sup_{\tau \in T} L_n(\tau)$$

and shows that under the null hypothesis (8):

$$\sup_{\tau \in T} L_n(\tau) \rightsquigarrow \sup_{\tau \in T} Q_1^2(\tau)$$

where  $Q_1(\cdot)$  is a Bessel process of order 1. Critical values for  $\sup Q_q^2(\cdot)$ , are extensively tabled in Andrews (1993).

### 3.3.2 Symmetry

In this subsection, we turn our attention to testing for asymmetric dynamics under the QAR framework. Thus we consider parameter constancy over  $\tau$  in representation (4):

$$\theta_j(\tau) = \beta_j, \quad \text{for all } \tau,$$

where  $\beta_j = E(\beta_{j,t})$ ,  $j = 1, \dots, p$ . This hypothesis can be formulated in the form of the null hypothesis:

$$\begin{aligned} H_0 &: R\theta(\tau) = r, \text{ with unknown but estimable } r. \\ \text{where, } R &= [0_{p \times 1} : I_p] \text{ and } r = [\beta_1, \beta_2, \dots, \beta_p]^T \end{aligned}$$

The Wald process and associated limiting theory provide a natural test for the hypothesis  $R\theta(\tau) = r$  when  $r$  is *known*. To test the hypothesis with *unknown*  $r$ , appropriate estimator of  $r$  is needed. In our econometric application, we consider the ordinary least square estimator. If we look at the process

$$\widehat{V}_n(\tau) = \sqrt{n} \left[ R\widehat{\Omega}_1^{-1}\widehat{\Omega}_0\widehat{\Omega}_1^{-1} \right]^{-\frac{1}{2}} \left( R\widehat{\theta}(\tau) - \widehat{r} \right)$$

then under  $H_0$ , Koenker and Xiao (2004a) proved that

$$\widehat{V}_n(\tau) \Rightarrow B_q(\tau) - f(F^{-1}(\tau)) [R\Omega_0^{-1}R^T] Z$$

where,  $Z = \lim \sqrt{n}(\widehat{r} - r)$ .

The necessity of estimating  $r$  introduces a drift component  $(f(F^{-1}(\tau)) [R\Omega_0^{-1}R^T] Z)$  in addition to the simple  $q$ -dimensional Brownian bridge process,  $B_q(\tau)$ , invalidating the distribution-free character of the original Kolmogorov-Smirnov (KS) test.

To restore the asymptotically distribution free nature of inference, Koenker and Xiao (2004a) employ a martingale transformation, proposed by Khmaladze (1981), over the process  $\widehat{V}_n(\tau)$ . In other words, Denote  $\frac{df(x)}{dx}$  as  $\dot{f}$  and define

$$\dot{g}(r) = \left( 1, \left( \frac{\dot{f}}{f} \right) (F^{-1}(r)) \right)^T,$$

and

$$C(s) = \int_s^1 \dot{g}(r) \dot{g}(r)^T dr.$$

We construct a martingale transformation  $K$  on  $\widehat{V}_n(\tau)$ , defined as:

$$\widetilde{V}_n(\tau) = K\widehat{V}_n(\tau) = \widehat{V}_n(\tau) - \int_0^\tau \left[ g_n(s)^T C_n^{-1}(s) \int_s^1 g_n(r) d\widehat{V}_n(r) \right] ds,$$

where  $g_n(s)$  and  $C_n(s)$  are uniformly consistent estimators of  $g(r)$  and  $C(s)$  over  $\tau \in T$ , and propose the following Kolmogorov-Smirnov type test based on the transformed process:

$$KH_n = \sup \| \widetilde{V}_n(\tau) \|.$$

Under the null hypothesis, the transformed process  $\widetilde{V}_n(\tau)$  converges to a standard Brownian motion.

Estimation of  $\Omega_0$  is straightforward:

$$\widehat{\Omega}_0 = \frac{1}{n} \sum_t x_t x_t^T$$

Koenker and Xiao (2004a) report to the works of Cox (1985) and Ng (1994) for estimation of  $\frac{\hat{f}}{f}$  and Koenker and Basset (1987), Koenker (1994) and Powell (1987) for estimation of  $\Omega_1$ .<sup>4</sup>

### 3.3.3 Local Sustainability

The concept of local sustainability is important to identify various dynamics (fiscal policies) compatible or not with public debt sustainability. It is useful to analyze the effect of fiscal misbehavior on long-run sustainability asking ourselves what would happen if some local dynamics were allowed to persist forever. Moreover, since the dynamic of the discounted U.S. Federal debt at each fixed quantile is represented by an autoregressive process, we could easily forecast debt values that should be avoided by policy makers if they were interested in keeping public debt sustainable. We believe that policy makers might be interested in using such forecasts as guidance instruments for monetary and fiscal policies.

**Local Stationarity.** In this subsection we focus on the analysis of local unit root behavior. As we have emphasized, the local behavior is analyzed in the QAR(p) framework. We express the local unit root hypothesis in the ADF representation (5) as:

$$H_0 : \alpha_1(\tau) = 1, \text{ for selected quantiles } \tau \in (0, 1)$$

For local stationarity tests, Koenker and Xiao (2004b) propose test similar to the conventional augmented Dick-Fuller (ADF) t-ratio test. The  $t_n$  statistics is the quantile autoregression counterpart of the ADF  $t$ -ratio test for a unit root and is given by:

$$t_n(\tau) = \frac{f(\widehat{F^{-1}}(\tau))}{\sqrt{\tau(1-\tau)}} (Y_{-1} P_X Y_{-1})^{\frac{1}{2}} (\hat{\alpha}_1(\tau) - 1),$$

where,  $f(\widehat{F^{-1}}(\tau))$  is a consistent estimator of  $f(F^{-1}(\tau))$ ,  $Y_{-1}$  is a vector of lagged dependent variables ( $y_{t-1}$ ) and  $P_X$  is the projection matrix onto the space orthogonal to  $X = (1, \Delta y_{t-1}, \dots, \Delta y_{t-p+1})$ .

Koenker and Xiao (2004b) shows that the limiting distribution of  $t_n(\tau)$  can be written as:

$$t_n(\tau) \Rightarrow \delta \left( \int_0^1 \underline{W}_1^2 \right)^{-\frac{1}{2}} \int_0^1 \underline{W}_1 dW_1 + \sqrt{1-\delta^2} N(0, 1)$$

where,  $\underline{W}_1(r) = W_1(r) - \int_0^1 W_1(s) ds$  and  $W_1(r)$  is a standard Brownian Motion.

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<sup>4</sup>This test for symmetry assumes that the process  $y_t$  is globally stationary. As we will see later, this condition is satisfied by the data used in this article.

Thus, the limiting distribution of  $t_n(\tau)$  is nonstandard and depend on parameter  $\delta$  given by:

$$\delta = \delta(\tau) = \frac{\sigma_{\omega\psi}(\tau)}{\sigma_{\omega}^2}.$$

Critical values for the statistic  $t_n(\tau)$  are provided by Hansen (1995, page 1155) for values of  $\delta^2$  in steps of 0.1. For intermediate values of  $\delta^2$ , Hansen suggest obtaining critical values by interpolation. We transcript the table of critical values in the Appendix.

We give the details on estimating nuisance parameters  $(\sigma_{\omega}^2, \sigma_{\omega\psi}(\tau))$  in the Appendix.

**Intercept Coefficient.** If the process were locally stationary and the null hypotheses of no local intercept is true, then we could say that the public debt is locally sustainable. In other words, if the fiscal policy (dynamic of the public debt) at a fixed quantile were to persist indefinitely, then public debt would ultimately converge to zero meaning that the PVBC condition would hold locally.

To test if the quantile intercept equals zero, we again follow Koenker e Machado (1999). We consider the quantile model representation (5).

Let  $\hat{\theta}(\tau)$  denote the minimizer of

$$\hat{V}(\tau) = \min_{\{\vartheta \in \mathbb{R}^{p+1}\}} \sum \rho_{\tau}(y_t - x_t^T \vartheta)$$

where  $x_t^T = (1, y_{t-1}, \Delta y_{t-1}, \dots, \Delta y_{t-p+1})^T$  and  $\tilde{\theta}(\tau)$  denote the minimizer for the corresponding constrained problem without intercept, with

$$\tilde{V}(\tau) = \min_{\{\vartheta \in \mathbb{R}^p\}} \sum \rho_{\tau}(y_t - x_{1t}^T \vartheta)$$

where  $x_{1t}^T = (y_{t-1}, \Delta y_{t-1}, \dots, \Delta y_{t-p+1})^T$ . Thus,  $\hat{\theta}(\tau)$  and  $\tilde{\theta}(\tau)$  denote the unrestricted and restricted quantile regression estimates.

Machado e Koenker(1999) states that we can test if we can statistically infer that the intercept is equal to zero using a related version of the Likelihood process for a quantile regression with the LR statistics given by (9).

Adapting the arguments of Koenker e Bassett (1982) slightly, it can be shown that under  $H_0$ ,  $L_n(\tau)$  is asymptotically  $X_q^2$  where  $q$  is the number of variables excluded in the restricted model. Since our impose the exclusion of only one term (the intercept) our critical value can be obtained from a Chi-squared table with one degree of freedom ( $q = 1$ ).

### 3.3.4 Global Sustainability

The concept of global sustainability states that episodes of fiscal imbalances resulting from government policies not compatible with long-run debt sustainability must be offset by periods of fiscal responsibility so that the PVBC condition holds in the long run. We next introduce tests for global stationarity and zero unconditional mean.

**Global Stationarity** An approach to test the unit root property is to examine the unit root property over a range of quantiles  $\tau \in T$ , instead of focusing only on a selected quantile. We may, then, construct a Kolmogorov-Smirnov (KS) type test based on the regression quantile process for  $\tau \in T$ .

Koenker and Xiao (2004b) consider  $\tau \in T = [\tau_0, 1 - \tau_0]$  for some small  $\tau_0 > 0$  to propose the following quantile regression based statistics for testing the null hypothesis of a unit root:

$$QKS = \sup_{\tau \in T} |U_n(\tau)|, \quad (10)$$

where  $U_n(\tau)$  is the coefficient based statistics given by:

$$U_n(\tau) = n(\hat{\alpha}_1(\tau) - 1).$$

Koenker and Xiao (2004b) suggest to approximate the limiting distribution of (10) under the null by using the autoregressive bootstrap (ARB). In this paper we will approximate the distribution under the null using the residual based block bootstrap procedure (RBB). The description of the RBB as well as its advantages over the ARB method are described in the Appendix

**Unconditional Mean Test** In order to test whether or not the unconditional mean of the process is zero, we recall that the following null hypotheses are equivalent:

$$\begin{aligned} H_0 &: \mu_y = 0 \\ H'_0 &: \mu_0 = 0 \end{aligned}$$

Consider the  $p$ th order quantile autoregressive process given by

$$\begin{aligned} y_t &= \theta_0(U_t) + \theta_1(U_t)y_{t-1} + \dots + \theta_p(U_t)y_{t-p} \\ &= \mu_0 + \beta_{1,t}y_{t-1} + \dots + \beta_{p,t}y_{t-p} + u_t \end{aligned}$$

where  $u_t = \theta_0(U_t) - \mu_0$ . Now note that the  $\tau$ th conditional quantile function of  $y_t$  is given by

$$Q_{y_t}(\tau | y_{t-1}, \dots, y_{t-p}) = \theta_0(\tau) + \theta_1(\tau)y_{t-1} + \dots + \theta_p(\tau)y_{t-p}$$

and it does not allow us to identify the intercept coefficient  $\mu_0$ , since  $Q_u(\tau) = \theta_0(\tau) - \mu_0$ , where  $\tau = Q_U(\tau)$  is the quantile function of  $U$ .

Thus, the next natural attempt would be to ignore the existence of asymmetric dynamic and estimate a symmetric regression (constant coefficient model)

$$y_t = \mu_0 + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} + v_t \quad (11)$$

The null hypothesis  $H'_0$  could be tested using the conventional t-statistics

$$t = \frac{\widehat{\mu_0}}{\widehat{SE(\mu_0)}}.$$

However, in omitting asymmetries, the new error term  $v_t$  is no longer an i.i.d. sequence, i.e.,

$$v_t = (\beta_{1,t} - \beta_1) y_{t-1} + \dots + (\beta_{p,t} - \beta_p) y_{t-p} + u_t,$$

which *invalids* the conventional t statistics type test.

Putting that aside, we decided to test directly the null hypothesis  $H_0 : \mu_y = 0$  using a resampling method for dependent data due to Carlstein (1986) named Nonoverlapping Block Bootstrap (NBB). The key feature of this bootstrap method is that its blocking rule is based on nonoverlapped segments of the data, making it able to simulate the weak dependence in the original series,  $y_t$ . A complete description of the NBB bootstrap is available in the appendix.

## 4 Results

For ease of comparison, we use an extension of Hamilton and Flavin data, provided and used by Davig (2004). The data set used comprises annual observations of discounted public debt over the period 1960 to 1998. This series is minutely adjusted to accurately represent the theoretical variables appearing in the present-value borrowing constraint<sup>5</sup>.

Figure 1 presents the discounted and undiscounted public debt series obtained using the Wilcox (1989) methodology. Observe that, although the undiscounted series appear not to be stationary, the discounted public debt series is much more well-behaved, with a clear decline in the 90's occurring during Clinton's term.

In addition, we can note sharp changes in the trend of the discounted public debt series in the years of 1974 and 1979, which are associated to the first and second oil crises. Even though these changes in fiscal policies have been object of concern on previous papers, they do not pose a threat to our experimental results, once our econometric model is robust to the presence of outliers.

### 4.1 Autoregressive Order Choice

We first determine the autoregressive order of the QAR(p) model (4) using the Kolmogorov-Smirnov test based on LR statistics. We start estimating the quantile regression below with  $p = p_{\max} = 3$ , that is:

$$Q_{y_t}(\tau \mid y_{t-1}, \dots, y_{t-p}) = \theta_0(\tau) + \theta_1(\tau) y_{t-1} + \theta_2(\tau) y_{t-2} + \theta_3(\tau) y_{t-3}.$$

The index set used is  $T = [0.1, 0.9]$  with steps of 0.005. Next, we test if the third order covariate is relevant in our model, i.e., we considered the null hypothesis:

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<sup>5</sup>Hamilton and Flavin (1986) provide a detailed discussion on the issue of adjusting official budget figures.



$$H_0 : \theta_3(\tau) = 0, \quad \text{for all } \tau \in T.$$

The results are reported in Table 1. Critical values were obtained in Andrews (1993). We can infer that the autoregressive variable  $y_{t-3}$  can be excluded from our econometric model.

Since the third order is not relevant, we proceed by analyzing if the second order covariate is relevant. Thus, we considered the null hypothesis:

$$H_0 : \theta_2(\tau) = 0,$$

whose results are presented in Table 2. Indeed, we verify that the second autoregressive variable cannot be excluded. Thus, the optimal choice of lag length in our model is  $p = 2$  and this order will be used in the subsequent estimation and hypothesis tests presented in this paper.

In summary, our econometric model will be:

$$y_t = \mu_0 + \beta_{1,t}y_{t-1} + \beta_{2,t}y_{t-2} + u_t$$

which has an associated ADF formulation as<sup>6</sup>:

$$y_t = \mu_0 + \alpha_{1,t}y_{t-1} + \alpha_{2,t}\Delta y_{t-1} + u_t.$$

The following conditional quantile function for  $y_t$  are generated from the aforementioned econometric model and its associated ADF formulation, that is:

$$Q_{y_t}(\tau \mid y_{t-1}, \dots, y_{t-p}) = \theta_0(\tau) + \beta_1(\tau)y_{t-1} + \beta_2(\tau)y_{t-2}, \quad (12)$$

$$Q_{y_t}(\tau \mid y_{t-1}, \dots, y_{t-p}) = \theta_0(\tau) + \alpha_1(\tau)y_{t-1} + \alpha_2(\tau)\Delta y_{t-1}. \quad (13)$$

Note that this result provides statistical evidence that the tests conducted in Davig (2004) may be misspecified, since the author assumes an autoregressive process with just one lag and our results suggest that the second order autoregressive covariate cannot be excluded. Moreover, the above model suggests that the autoregressive coefficients,  $\alpha_{1,t}$  and  $\alpha_{2,t}$ , may be random.

## 4.2 Symmetry

We apply the symmetry test described in last section to verify if the data really provides statistical evidence of asymmetric dynamic. We first obtain the OLS estimates for  $\beta_1$  and  $\beta_2$ ,  $\beta_{1,OLS}$  and  $\beta_{2,OLS}$ , respectively. So, we want to test the null  $\beta_1(\tau) = \beta_{1,OLS}$  and  $\beta_2(\tau) = \beta_{2,OLS}$  in equation (12) for all  $\tau \in [0.05, 0.95]$  over steps of 0.005. We then employ the martingale transformation on the Kolmogorov-Smirnov test statistic. Following Koenker and Xiao (2004a) we used the rescaled Bofinger (1975) bandwidth  $0.6h_B$  in estimating the sparsity

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<sup>6</sup>For the sake of completion, we carried on the same tests in the ADF form. As expected, the Kolmogorov-Smirnov based on LR statistics estimates were exactly the same as the estimates reported in Tables 1 and 2.

function. The values of the test statistics are given in Table 3 with critical values coming from Koenker and Xiao (2002). To test for each parameters isolatedly using a closure of  $[0.05, 0.95]$  the reported 5% critical value is 2.140. Therefore, the empirical results indicate that asymmetric behavior exist in the discounted U.S. federal debt series.

### 4.3 Local sustainability

To obtain a detailed description on the discounted public debt dynamic, we first examine the unit root behavior at various quantiles in the proposed QAR model by using the t-ratio test  $t_n(\tau)$ . Table 4 reports the results. The second column presents the estimate of the autoregressive root at each decile. Note that  $\hat{\alpha}_1(\tau)$  decreases when we move from lower quantiles to higher quantiles, suggesting that the discounted U.S. public debt is more close to nonstationarity at low quantiles. The third and forth columns of Table 4 report the calculated t-statistic  $t_n(\tau)$  and  $\delta^2$  parameter. The majority results reject the null hypothesis of  $H_0 : \alpha_1(\tau) = 1$  against the alternative hypothesis  $H_1 : \alpha_1(\tau) < 1$ . The Critical values were obtained by interpolation of the critical values appearing in Table 11 extracted from Hansen (1995, page 1155). Note, however, that the null hypothesis of unit root cannot be rejected at low deciles. Hence, the results for local stationarity suggest that when the discounted public debt attains a sufficient high level, the U.S. government intervenes in order to push the public debt to a stationary path. This finding is not totally new in the literature: Arestis et al.(2004) reported similar results employing a threshold autoregressive model estimated with budget deficit data. Thus, in using an econometric model robust against distribution misspecification and outlier observations, we confirm results previously reported in the literature.

Table 5 presents the results for the unconditional mean test. The third and forth columns report the restricted and unrestricted minimum value estimates for the discounted public debt series. The fifth column report the log-likelihood based statistic estimates, which must be compared to 3.84 to perform a  $\chi_1^2$  test with significance level of 5%.

We can see that only for quantiles around median can the null hypothesis  $H_0 : \theta_0(\tau) = 0$  not be rejected. At both low quantiles and high quantiles the intercept coefficient condition is rejected. As previously observed, long-run sustainability of the public debt requires that the discounted U.S. federal debt series is both stationary and satisfies the intercept coefficient condition. When we combine in Table 6 the results obtained in Table 4 and 5, we find that the public debt series possesses three different regimes according to the level of debt:

1. at low quantiles (quantiles 0.1 to 0.3) the debt series is nonstationary and does not satisfy the intercept coefficient condition, meaning that its dynamics is unsustainable. We notice a negative point estimate for the intercept that, together with nonstationarity, suggests that if government keeps public debt too low for long, then ultimately the public debt would diverge to minus infinity, meaning not only that transversality condition

would be violated but also the existence of an inefficient taxation scheme, in the sense that the deadweight cost of taxation would be too large for negative values of debt.<sup>7</sup> Therefore, the U.S. government should eventually move away from this local fiscal policy if it is interested in avoiding inefficient taxation and unsustainable trajectories of the public debt;

2. medium values of the public debt (quantiles 0.4 to 0.7) form an area of sustainability, in which the discounted debt is a zero-mean stationary process;
3. at high quantiles (quantiles 0.8 and 0.9), the discounted public debt follow a stationary path, but it does not converge to zero (does not satisfy the intercept condition). This may suggest that for high values of public debt, the political incentives are enough to make the government decide for a stationary process but budgetary cuts sufficient to guarantee convergence to zero may be too costly. Again, this specific fiscal policy should be avoided by the U.S. government.

Hence, these findings suggest the existence of a band of sustainability, but also indicate that there are episodes in which the U.S. public debt moves out of the band. We use the quantile autoregressive model to make in-sample forecasts of the public debt for  $\tau = 0.3$  and 0.8. In other words, we want to estimate the upper and lower bounds of the sustainability band in order to identify the episodes during which public debt jumped out of the band. Figures 2 and 3 present the in-sample forecast for the 0.3th and 0.8th conditional quantile of the discounted public debt, respectively. Such trajectories provide a lower and upper bound for the band of sustainability. Note in these figures that the original data series of discounted public debt does not lie inside the band in about 16 years.

Table 7 reports the events associated to public debt levels larger than the upper bound. There are four important events: years following first and second oil shocks, recession beginning of Bush's term, and Gulf war. This result shows that the occasional periods of expanding public debt in the U.S. are mostly motivated by unpredictable events like oil shocks and wars. Table 8 reports the events associated to public debt levels less than the lower bound. We notice that most of the periods during which U.S. debt moves beyond its lower bound are associated to episodes of persistent and strong fiscal adjustment, as in Carter's term, the Tax Reform Act of 1986 and Emergency Deficit Control Act, and the Clinton's second term. This suggests that the persistent decreasing in the US public debt observed during periods of fiscal reforms stops when the public debt reaches a very low level deemed to be economically inefficient.

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<sup>7</sup>This argument is based on the fact that the taxation involves not only one-to-one transfer of purchasing power from individuals to the government but also some collection costs and/or indirect misallocation costs that are imposed on the private economy. Therefore, the production of government revenue involves the using up of some resources in the sense of costs that are referred to as deadweight losses (Barro, 1979).

#### 4.4 Global sustainability

We calculated both the QKS statistics and its critical value maximizing (7) over the set  $T = [0.1, 0.9]$  with steps of 0.005. The critical values were obtained using the residual-based block bootstrap recently proposed by Paparodis and Politis (2003). Table 9 reports the statistics and critical value for three different values of the block length,  $b$ , arbitrarily chosen. We considered 10,000 bootstrap replications

Note that the fundamental issue of the RBB bootstrap is its ability to simulate the weak dependence appearing in the original data series by separating the residuals in blocks. As the block length increases, the simulated dependence in the pseudo-series becomes more accurate (see appendix for details).

There is evidence that the discounted US federal debt is not a unit root process with significance level of 5% for almost all values of  $b$  (except for  $b = 8$ , where we reject the unit root null at significance level of 10%). Thus, the results in Table 9 suggest that the discounted U.S. federal debt is globally stationary.

We now test the null hypothesis that the discounted debt process has zero unconditional mean,  $H_0 : \mu_y = 0$ . Recall that testing such hypothesis correspond to testing the null  $\mu_0 = E[\theta_0(U_t)] = 0$ . Notice that in our local analysis we tested the null  $\theta_0(\tau) = 0$  for various fixed quantiles. In particular, results from Table 5 show that we cannot reject the null  $\theta_0(0.5) = 0$ . Hence, under the courageous assumption that the distribution of  $\theta_0(U_t)$  is symmetric, not to reject  $\theta_0(0.5) = 0$  would be equivalent to not to reject  $\mu_0 = 0$ . However, the distribution of  $\theta_0(U_t)$  is probably skewed but, even so, the results in Table 5 still provide some evidence that  $\mu_0 = 0$  since we cannot reject the null  $\theta_0(\tau) = 0$  for various quantiles around and including the median, that is,  $\tau = 0.3, 0.4, 0.5, 0.6, 0.7$ . Nevertheless, the distribution of  $\theta_0(U_t)$  may be highly skewed and therefore we still need a formal test. We conduct a t-test for the unconditional mean and use the NBB resampling method with 10,000 replications to compute 5% critical values. Table 10 reports the t-statistic for the discounted public debt series. The reported results suggest that the unconditional mean of the autoregressive process is not statistically different from zero. This result associated with the QKS result for global stationarity present evidence that the public debt is globally sustainable.

Putting all together, the discounted U.S. federal debt is globally sustainable despite the fact that local unsustainability can be found at some fixed quantiles  $\tau$ . In other words, episodes in which discounted public debt is too high or too low not seem to persist forever and are offset by many other episodes of fiscal responsibility that makes discounted U.S. federal debt be globally sustainable in the long run.

### 5 Conclusion

In this work, we have empirically explored the question of whether the fiscal policy in the United States is sustainable in the long-run using data on discounted

public debt for the period from 1960 to 1998. The theoretical framework has been provided by the present value borrowing constrain that allows for a stochastic real interest rate as in Wilcox (1989) and Davig (2004). Following recent econometric studies that suggest the existence of regime shifts of fiscal policy (Davig, 2004; Arestis et al. 2004), we use a quantile autoregression model recently proposed by Koenker and Xiao (2004a, 2004b) to test if the data provides evidence of asymmetries in the U.S. public debt.

Our econometric model accounts for many regime shifts and possesses robustness against distribution misspecification and outlier observations. We confirm previous results in the literature concerning the existence of asymmetric dynamic in the U.S. public debt and global sustainability. As for local sustainability we report new results. In particular, three regimes of fiscal policy were identified: (1) when the public debt is low, fiscal policy is unsustainable; (2) for values of the public debt around (and including) median, fiscal policy is sustainable characterizing what we called band of sustainability; (3) when public debt attains high values, fiscal policy is unsustainable, even though the discounted public debt follow a stationary path.

We used the local dynamics to make in-sample forecast in order to identify episodes in which the debt moved out of the band of sustainability. It is shown that periods in which the debt reach the band upper bound are associated to wars and oil shocks. On the other hand, the debt reaches the band lower bound specially during periods of fiscal reforms. We stress the fact that we could use the local dynamics to make out-of-sample forecasts of the the public debt and use it to construct a measure of indebtedness compatible with long-run sustainability. In the same way as inflation target measures, we believe that such forecasts might be useful as guidance instruments for monetary and fiscal policies. From the theoretical econometricians' perspective, this paper can be regarded as an initial attempt to apply the QAR process to the empirical analysis of public debt sustainability. Further theoretical developments of economic models that could be used to explain the existence of a band of sustainability, in addition to further empirical applications, would be very fruitful.

## List of Tables

**Table 1: Results for the autoregressive order choice test excluding variable  $y_{t-3}$ .**

excluded variable	$\sup_{\tau \in T} L_n(\tau)$ estimate	5% critical value	10% critical value	$H_0 :$ $\theta_3(\tau) = 0$
$y_{t-3}$	3.31	9.31	7.36	<i>do not reject</i>

**Table 2: Results for the autoregressive order choice test excluding variable  $y_{t-2}$ .**

excluded variable	$\sup_{\tau \in T} L_n(\tau)$ estimate	5% critical value	10% critical value	$H_0 :$ $\theta_2(\tau) = 0$
$y_{t-2}$	30.90	9.31	7.36	<i>reject</i>

**Table 3: Results for the Symmetry test.**

Coefficient on	test statistic	5% critical value	<i>OLS</i> estimate of $\beta_j$	$H_0 :$ $\beta_j(\tau) = \beta_{j,OLS}$
$y_{t-1}$	4.190	2.140	1.56	<i>reject</i>
$y_{t-2}$	3.123	2.140	-0.63	<i>reject</i>

**Table 4 : local stationarity results with t-statistics  $t_n(\tau)$ .**

$\tau$	$\hat{\alpha}_1(\tau)$	$t_n(\tau)$	$\delta^2$	$H_0 : \alpha_1(\tau) = 1$
<b>0.1</b>	1.039	2.998	0.051	<i>Do not reject at 5%</i>
<b>0.2</b>	1.031	3.952	0.184	<i>Do not reject at 5%</i>
<b>0.3</b>	0.997	-0.252	0.107	<i>Do not reject at 5%</i>
<b>0.4</b>	0.961	-3.842	0.111	<i>reject at 5%</i>
<b>0.5</b>	0.950	-4.786	0.098	<i>reject at 5%</i>
<b>0.6</b>	0.943	-4.795	0.140	<i>reject at 5%</i>
<b>0.7</b>	0.879	-7.884	0.169	<i>reject at 5%</i>
<b>0.8</b>	0.863	-10.456	0.394	<i>reject at 5%</i>
<b>0.9</b>	0.863	-7.893	0.112	<i>reject at 5%</i>

**Table 5: Results for intercept coefficient test.**

$\tau$	$\hat{\mu}_0(\tau)$	$\mathbf{L}_n(\tau)$	$\mathbf{H}_0 : \hat{\mu}_0(\tau) = 0$
<b>0.1</b>	-43226.9	6.632	<i>reject at 5%</i>
<b>0.2</b>	-35994.42	5.061	<i>reject at 5%</i>
<b>0.3</b>	-12482.67	0.649	<i>Do not reject at 5%</i>
<b>0.4</b>	12548.71	0.282	<i>Do not reject at 5%</i>
<b>0.5</b>	20997.99	0.991	<i>Do not reject at 5%</i>
<b>0.6</b>	26649.5	1.912	<i>Do not reject at 5%</i>
<b>0.7</b>	67595.38	3.820	<i>Do not reject at 5%</i>
<b>0.8</b>	89075.46	10.163	<i>reject at 5%</i>
<b>0.9</b>	96941.13	13.741	<i>reject at 5%</i>

**Table 6: Summary of results for local sustainability test.**

$\tau$	Starionarity	Zero Un-conditional Mean	Sustainability
<b>0.1</b>	-	-	-
<b>0.2</b>	-	-	-
<b>0.3</b>	-	Ok	-
<b>0.4</b>	Ok	Ok	Ok
<b>0.5</b>	Ok	Ok	Ok
<b>0.6</b>	Ok	Ok	Ok
<b>0.7</b>	Ok	Ok	Ok
<b>0.8</b>	Ok	-	-
<b>0.9</b>	Ok	-	-

**Table 7: years when the discounted public debt in larger than the 0.8th conditional quantile forecast and associated political/economic event.**

Year	Economic Event
1975	right after first oil crisis
1981 and 1983	right after second oil crisis
1988	Recession beginning of Bush's term
1991-92	Gulf War

Table 8: years when the discounted public debt is lower than the 0.3th conditional quantile forecast and associated political/economic event.

Year	Economic Event
1965 and 1969	Johnson's term
1973-74	Nixon's term
1977-79	Carter's term
1987	Tax Reform Act of 1986 and Emergency Deficit Control Act
1997-98	Clinton's second term

Table 9: Results for the global stationarity test.

Block length $b$	$QKS$	5% critical value	10% critical value	$H_0 : \alpha_1 = 1$
8	6.3139	6.4807	5.6270	<i>reject at 10%</i>
12	6.3139	5.9743	5.1330	<i>reject at 5%</i>
16	6.3139	5.8237	5.1770	<i>reject at 5%</i>

Table 10: results for the unconditional mean test.

$t$	2.5% <i>critical value</i>	97.5% <i>critical value</i>	$H_0 : \text{intercept} = 0$
$1,45 \cdot 10^{-5}$	$-1,84 \cdot 10^{-5}$	$9,36 \cdot 10^{-5}$	<i>do not reject at 5%</i>

Table 11: Asymptotic critical values of the t-statistic  $t_n(\tau)$

$\delta^2$	1%	5%	10%
0.9	-3.39	-2.81	-2.50
0.8	-3.36	-2.75	-2.46
0.7	-3.30	-2.72	-2.41
0.6	-3.24	-2.64	-2.32
0.5	-3.19	-2.58	-2.25
0.4	-3.14	-2.51	-2.17
0.3	-3.06	-2.40	-2.06
0.2	-2.91	-2.28	-1.92
0.1	-2.78	-2.12	-1.75

## 6 Appendix

### 6.1 Estimation of nuisance parameters

Machado and Koenker (1999) note that "It is a somewhat unhappy fact of life that the asymptotic precision of quantile estimates in general, and quantile



regression estimates in particular, depend on the reciprocal of a density function evaluated at the quantile of interest - a quantity Tukey (1965) termed the 'sparsity function' and Parzen (1979) calls the quantile-density function. It is perfectly natural that the precision of quantile estimates should depend on this quantity, because it reflects the density of observations near the quantile of interest. If the data are very sparse at the quantile of interest, then it will be difficult to estimate. On the other hand, when the sparsity is low, so that observations are very dense, the quantile will be more precisely estimated." Thus, to estimate the precision of the  $\tau$ th quantile regression estimate directly, the nuisance quantity

$$s(\tau) = [f(F^{-1}(\tau))]^{-1}$$

must be estimated. To estimate  $s(\tau)$  in the one-sample model we follow Sidiqui's idea of differentiating the identity  $F(F^{-1}(t)) = t$ . We find that the sparsity function is simply the derivative of the quantile function; that is

$$\frac{d}{dt}F^{-1}(t) = s(t).$$

So, just as differentiating the distribution function  $F$  yields the density function  $f$ , differentiating the quantile function  $F^{-1}$  yields the sparsity function  $s$ . Therefore, to estimate  $s(t)$  we use a simple difference quotient of the empirical quantile function; that is,

$$\hat{s}_n(t) = \frac{\hat{F}_n^{-1}(t + h_n) - \hat{F}_n^{-1}(t - h_n)}{2h_n}.$$

where  $\hat{F}_n^{-1}$  is an estimate of  $F^{-1}$  and  $h_n$  is a bandwidth that may tend to 0 as  $n \rightarrow \infty$ . A bandwidth rule proposed by Bofinger (1975) is derived based on minimizing the mean squared error of the density estimator and is of order  $n^{-\frac{1}{5}}$ :

$$h_B = n^{-\frac{1}{5}} \left[ \frac{4.5s^2(t)}{(s''(t))^2} \right]^{\frac{1}{5}}.$$

In the absence of information about the form of  $s(\cdot)$ , we plug-in the Gaussian model to select bandwidth and obtain

$$h_B = n^{-\frac{1}{5}} \left[ \frac{4.5\phi^4(\Phi^{-1}(t))}{(2(\Phi^{-1}(t))^2 + 1)^2} \right]^{\frac{1}{5}}.$$

Koenker and Xiao (2004a) Monte Carlo results indicate that the Bofinger bandwidth provides reasonable upper bound for bandwidth selection in testing parameter constancy. Thus, they suggest the utilization of a rescaled version of Bofinger bandwidth ( $h = 0.6h_B$ ).

One way of estimating  $F^{-1}(s)$  is to use a variant of the empirical quantile function for the linear model proposed in Bassett and Koenker (1982),

$$\hat{Q}(\tau | \bar{x}) = \bar{x}^T \hat{\alpha}(\tau).$$

In summary,  $f(F^{-1}(t))$  can be estimated by

$$f_n(F_n^{-1}(t)) = \frac{1}{\widehat{s}_n(t)} = \frac{1.2h_B}{\bar{x}^T(\widehat{\alpha}(\tau + 0.6h_B) - \widehat{\alpha}(\tau - 0.6h_B))}.$$

For the long-run variance and covariance parameters, we may use the kernel estimators

$$\begin{aligned}\sigma_\omega^2 &= \sum_{h=-M}^M k\left(\frac{h}{M}\right) C_{\omega\omega}(h) \\ \sigma_{\omega\psi}(\tau) &= \sum_{h=-M}^M k\left(\frac{h}{M}\right) C_{\omega\psi}(h)\end{aligned}$$

where  $u_{t\tau} = y_t - x_t^T \alpha(\tau)$ ,  $\psi_\tau(u) = \tau - I(u < 0)$  and  $\omega = \Delta y_t$ . We choose to use the lag window

$$k\left(\frac{h}{M}\right) = 1 - \frac{h}{1+M},$$

but what it is necessary is that  $k(\cdot)$  is defined on  $[-1, 1]$  with  $k(0) = 1$ . The bandwidth (truncation) parameter  $M$  used was

$$M = \text{integer} \left[ 4 \sqrt[3]{\frac{n}{100}} \right].$$

The quantities  $C_{\omega\omega}(h)$  and  $C_{\omega\psi}(h)$  are simply the auto sample covariance of  $\omega$  and sample covariance between  $\omega$  and  $\psi$ , respectively, i.e.:

$$\begin{aligned}C_{\omega\omega}(h) &= \frac{1}{n} \sum \omega_t \omega_{t+h} \\ C_{\omega\psi}(h) &= \frac{1}{n} \sum \omega_t \psi_\tau(\widehat{u}_{(t+h)\tau})\end{aligned}$$

## 7 The Non-overlapping Block Bootstrap (NBB)

The NBB procedure can be summarized as follows:

Let  $\widehat{\mu}_y$  denote the sample mean of initial data series and suppose that  $l \equiv l_n \in [1, n]$  is an integer and  $b \geq 1$  is the largest integer satisfying  $lb \leq n$ . Then, define the nonoverlapping blocks

$$\mathfrak{B}_i = (y_{(i-1)l+1}, \dots, y_{il})^T, \quad i = 1, \dots, b.$$

Step 1: Select a random sample of blocks  $\mathfrak{B}_1^*, \dots, \mathfrak{B}_k^*$  with replacement from  $\{\mathfrak{B}_1, \dots, \mathfrak{B}_b\}$  for some suitable integer  $k > 1$ .

Step 2: With  $m = kl$ , let  $\mu_{m,n}^*$  denote the empirical estimate of the sample mean of the bootstrap sample  $y^* \equiv (y_1^*, \dots, y_l^*; \dots; y_{\{(b-1)l+1\}}^*, \dots, y_m^*)$  obtained by writing the elements of  $\mathfrak{B}_1^*, \dots, \mathfrak{B}_k^*$  in a sequence, i. e.:

$$\mu_{m,n}^* = \frac{1}{m} \sum_{j=1}^m y_j^*.$$

Repeating steps 1-2 a great number of times ( $BB$ ) we obtain a collection of pseudo-statistics  $\mu_{m,n}^{*(1)}, \dots, \mu_{m,n}^{*(BB)}$ . An empirical distribution based on the pseudo-statistics  $\mu_{m,n}^{*(1)}, \dots, \mu_{m,n}^{*(BB)}$  provide a consistent approximation of the distribution of  $\hat{\mu}_y$ . Let  $C_t^* \left( \frac{\lambda}{2} \right)$  and  $C_t^* \left( 1 - \frac{\lambda}{2} \right)$  be the  $(100 \frac{\lambda}{2})$ th and  $(100 (1 - \frac{\lambda}{2}))$ th quantiles of the empirical distribution of  $\mu_{m,n}^*$  respectively, i.e.

$$P^* \left( \mu_{m,n}^* \leq C_t^* \left( \frac{\lambda}{2} \right) \right) = \frac{\lambda}{2}$$

and

$$P^* \left( \mu_{m,n}^* \leq C_t^* \left( 1 - \frac{\lambda}{2} \right) \right) = 1 - \frac{\lambda}{2}.$$

Then the null hypothesis will be rejected at the  $(1 - \lambda)$  level if

$$\hat{\mu}_y \leq C_t^* \left( \frac{\lambda}{2} \right) \quad \text{or} \quad \hat{\mu}_y \geq C_t^* \left( 1 - \frac{\lambda}{2} \right).$$

## 7.1 Residual-Based Block Bootstrap for Unit Root Testing

This subsection describes the resampling method for detecting the presence of unit root in time series proposed by Paparoditis and Politis (2003).

For testing whether or not a series exhibit nonstationary behavior, the common assumption is that a time series  $\{y_t\}$  is either stationary around a (possibly nonzero) mean, or I(1), i.e., integrated of order one; as usual, the I(1) condition means that  $\{y_t\}$  is not stationary, but its first difference series is stationary. The hypothesis test setup can then be stated as:

$$\begin{aligned} H_0 &: \{y_t\} \text{ is I(1)} && \text{versus} && (14) \\ H_1 &: \{y_t\} \text{ is stationary.} \end{aligned}$$

The authors point out that "the bootstrap procedure should be able to reproduce the sampling distribution of the test statistic under the null hypothesis *whether the observed series obeys the null hypothesis or not*. (...) it is not sufficient to be able to generate unit root pseudo-data, given unit root data; the successful procedure must be able to generate unit-root pseudo-data even if the true data happen to be stationary. This point has not been appropriately taken into account in the literature where bootstrap approaches are applied to differenced observations and/or the theory of bootstrap validity is often derived under the assumption that the observed process is unit root integrated. Furthermore, applying the block bootstrap to the differenced series fails if the null hypothesis is wrong, i.e., the corresponding bootstrap statistic diverges to minus infinity, leading to a loss of power. "

On carrying out the hypothesis testing (14) it is necessary to choose a parameter  $\alpha$  with the property that  $\alpha = 1$  is equivalent to  $H_0$ , whereas  $\alpha \neq 1$

is equivalent to  $H_1$ . Once decided which parameter  $\alpha$  to use, Paparoditis and Politis (2003) suggests defining a new series  $\{U_t\}$  as:

$$U_t = (y_t - \beta - \alpha y_{t-1})$$

for  $t = 1, 2, \dots$  where the constant  $\beta$  is defined by  $\beta = E(y_t - \alpha y_{t-1})$  so that  $E(U_t) = 0$ . This new series is very useful because is stationary *always*: under  $H_0$  and/or  $H_1$ .

In order to make clear how the RBB algorithm works, lets assume we want to carry out the traditional ADF. First, consider the ADF equation below

$$\Delta y_t = \alpha y_{t-1} + \sum_{j=1}^p \alpha_{j+1} \Delta y_{t-j} + u_t \quad (15)$$

and let  $\hat{\alpha}_n$  be the OLS estimator of  $\alpha$ . Next, define the centered differences

$$D_t = y_t - y_{t-1} - \frac{1}{n-1} \sum_{s=2}^n (y_s - y_{s-1}).$$

These differences are the bootstrap analogous to the differences of the original series  $\{y_t\}$  appearing in the ADF regression (15). We now describe the RBB bootstrap algorithm applied to the ADF case.

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Step 1: first calculate the centered residuals

$$\hat{U}_t = (y_t - \tilde{\alpha}_n y_{t-1}) - \frac{1}{n-1} \sum_{s=2}^n (y_s - \tilde{\alpha}_n y_{s-1}), \quad \text{for } t = 2, 3, \dots, n$$

where  $\tilde{\alpha}_n$  is any consistent estimator of  $\alpha$  based on the observed data  $\{y_1, y_2, \dots, y_n\}$ .

Step 2: Choose a positive integer  $b (< n)$ , and let  $i_0, i_1, \dots, i_{k-1}$  be drawn i.i.d with distribution uniform on the set  $\{1, 2, \dots, n-b\}$ ; where  $k = \left\lceil \frac{(n-1)}{b} \right\rceil$ , and  $[\cdot]$  denotes the integer part of the number  $\cdot$ . The procedure constructs a bootstrap pseudo-series  $y_1^*, \dots, y_l^*$ , where  $l = kb + 1$ , as follows:

$$y_t^* = \begin{cases} y_1 & \text{for } t = 1, \\ \hat{\beta} + y_{t-1}^* + \hat{U}_{i_m+s} & \text{for } t = 2, 3, \dots, l. \end{cases}$$

where

$$\begin{aligned} m &= \left\lceil \frac{(t-2)}{b} \right\rceil \\ s &= t - mb - 1 \end{aligned}$$

and  $\hat{\beta}$  is a drift parameter that is either set equal to zero or is a  $\sqrt{n}$ -consistent estimator of  $\beta$ .

Additionally, the procedure also generate a pseudo-series of  $l$  centered differences denoted by  $D_1^*, D_2^*, \dots, D_l^*$  as follows: For the first block of  $b + 1$  observations we set  $D_1^* = 0$  and

$$D_j^* = D_{i_0+j-1} \quad \text{for } j = 2, 3, \dots, b + 1.$$

For the  $(m + 1)$ th block,  $m = 1, \dots, k - 1$  we define

$$D_{mb+1+j}^* = D_{i_m+j}, \quad \text{where } j = 1, 2, \dots, b.$$

Step 3: We then calculate the regression of  $y_t^*$  on  $y_{t-1}^*$  and on  $D_{1-1}^*, D_{t-2}^*, \dots, D_{t-p}^*$ . Use the least squares estimator of the coefficient on  $y_{t-1}^*$  to compute the pseudo-statistic  $\hat{\alpha}^*$ .

Step 4: repeating steps 2-3 a great number of times ( $BB$ ) we obtain a collection of pseudo-statistics  $\hat{\alpha}_1^*, \dots, \hat{\alpha}_{BB}^*$ . An empirical distribution based on the pseudo-statistics  $\hat{\alpha}_1^*, \dots, \hat{\alpha}_{BB}^*$  provide a consistent approximation of the distribution of  $\hat{\alpha}_n$  under the null hypothesis  $H_0 : \alpha = 1$ . The  $\tau$ -quantile of the bootstrap distribution in turn yields a consistent approximation to the  $\tau$ -quantile of the true distribution (under  $H_0$ ), which is required in order to perform a  $\tau$ -level test for  $H_0$ .

#### THE RBB TESTING ALGORITHM TO KOLMOGOROV-SMIRNOV TEST OF GLOBAL STATIONARITY

To resample the limiting distribution of  $QKS$  given by (10), we follow steps 1 and 2 in the above procedure and replace steps 3 and 4 by

Step 3': We then calculate the quantile regression of  $y_t^*$  on  $y_{t-1}^*$  and on  $D_{1-1}^*, D_{t-2}^*, \dots, D_{t-p}^*$ . Use the quantile estimator of the coefficient on  $y_{t-1}^*$  to compute the pseudo-statistic  $\hat{\alpha}^*(\tau)$ .

Step 4': repeating steps 2-3 a great number of times ( $BB$ ) we obtain a collection of pseudo-statistics  $\hat{\alpha}_1^*, \dots, \hat{\alpha}_{BB}^*$ . We then generate the pseudo-statistics

$$QKS_i^* = \sup_{\tau \in T} |n(\hat{\alpha}_i^*(\tau) - 1)|, \quad \text{for } i = 1, \dots, BB.$$

An empirical distribution based on the pseudo-statistics  $QKS_1^*, \dots, QKS_{BB}^*$  provide a consistent approximation of the distribution of  $QKS$  under the null hypothesis  $H_0 : \alpha = 1$ . The  $\tau$ -quantile of the bootstrap distribution in turn yields a consistent approximation to the  $\tau$ -quantile of the true distribution (under  $H_0$ ), which is required in order to perform a  $\tau$ -level test for  $H_0$ .

Our choice of using the RBB procedure instead of the resampling procedure proposed in Koenker and Xiao (2004b) based on difference of original series was justified by the belief that the latter have low power, i.e., the experimental distribution does not represent the null hypothesis when the real data are generate by a stationary process.

The great advantage of the RBB process is that the algorithm manage to automatically (and nonparametrically) replicate the important weak dependence characteristics of the data, e.g., the dependence structure of the stationary process  $\{y_t\}$  and at the same time to mimic correctly the distribution of a particular test statistic under the null.

There is still the problem of how to choose the parameter  $b$  of block length. This parameter is of great importance to the Residual-based block bootstrap procedure to simulate weak dependence in the pseudo-series and thus providing power to the test. Observe in Step 2, that only one observation of each block is obtained as an aleatory drawn of the centered residual's set with the others having centered residuals associated to the aleatory drawn.

The asymptotic results hold true for any block size satisfying

$$b \rightarrow \infty \quad \text{but} \quad \frac{b}{\sqrt{n}} \rightarrow 0.$$

Nevertheless, there is still many choice of  $b$  that satisfy the above condition, which leads to the natural question of what is the optimal choice of  $b$ . Unfortunately, this question remains unanswered. All methods described in the literature are somehow heuristic. Paparoditis and Politis (2003) note that "more work is required in order to give analytical and/or empirical substantiation to the block size choice ideas".

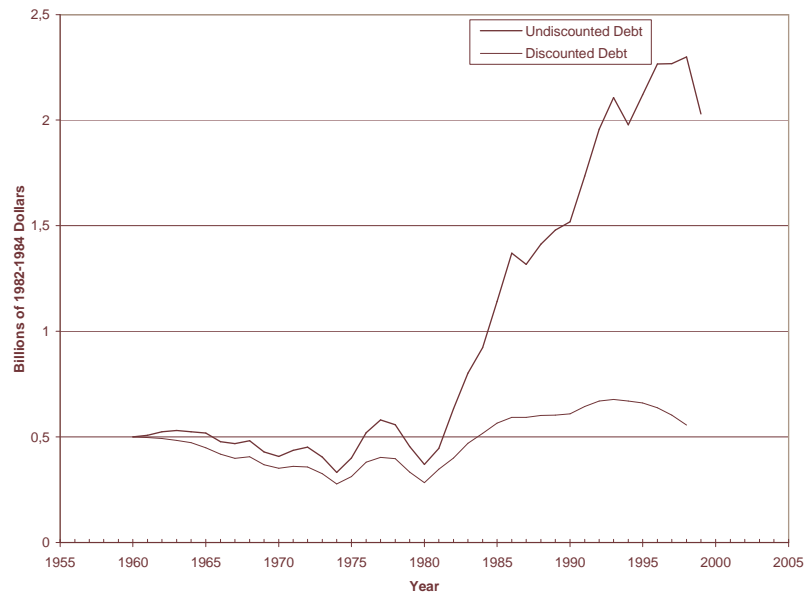


Figure 1: Discounted and Undiscounted Public Debt Series

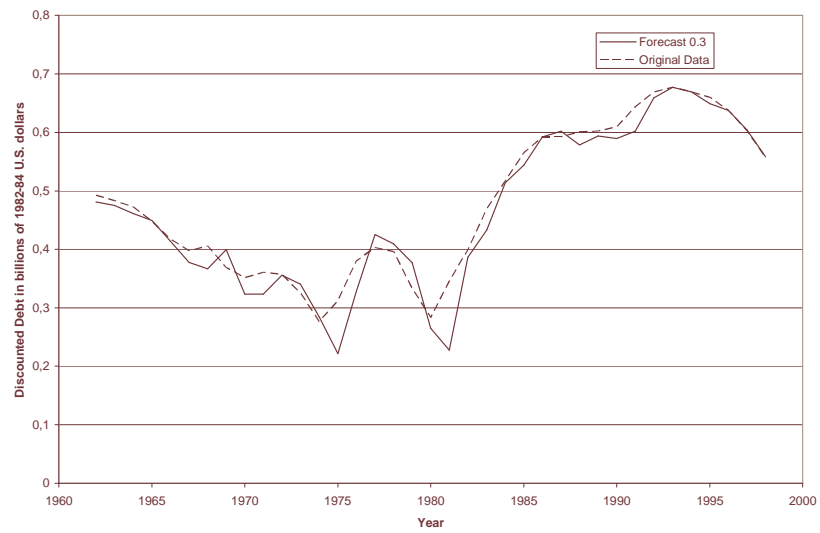


Figure 2: Discounted public debt series and 0.3th conditional quantile in-sample forecast.

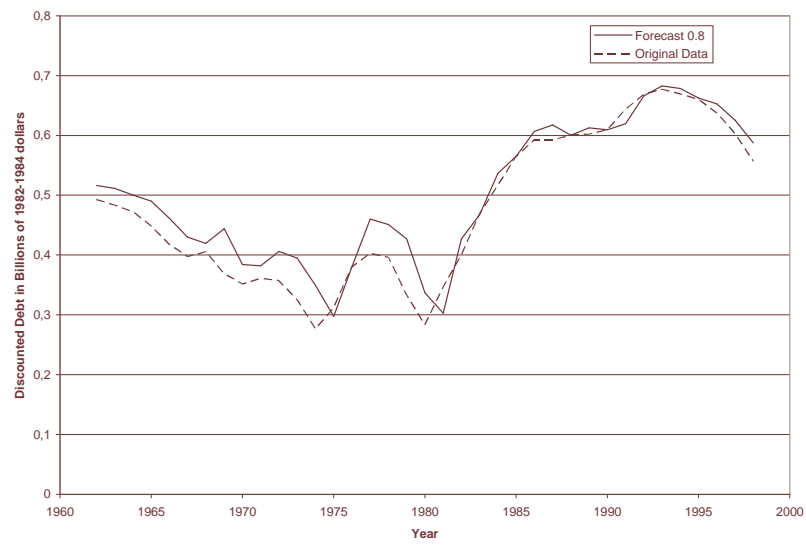


Figure 3: Discounted public debt series and 0.8th conditional quantile in-sample forecast.



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